Mechanics and Ballistics of Pneumatic Spud Cannons

The Mathcad file for this paper is available at: **http://vxphysics.com/Mathcad/**

The Purpose of this paper is to model the mechanics and ballistics of a pneumatic spud cannon.

We will do this in this 5 steps:

Description of a Spud Cannon and Definition of Terms
Develop models, equations, and methodology for determining muzzle velocity and trajectory
Comparison of Velocities and Accelerations of Different Models of Internal Muzzle Ballistics
Comparison between Models and Performance Data
Implementation of some ways to better model internal muzzle behavior

This paper offers an examination of the fundamental parameters that govern the internal ballistics of a cannon's muzzle velocity, as well as the resulting trajectory of the projectile in flight. Beginning with a detailed lexicon of terms and values in Section I, we establish a foundation for the complex interplay between internal geometry, volume, pressure, the force of barrel resistance, loss mechanisms and the resulting acceleration and spud velocity within the barrel of the cannon.

We then delve into the application of an Energy Balance Model (EBM) in Section II to capture the incremental changes in muzzle velocity over minute distances. This model forms the basis for the computational tool outlined in Section III, which is utilized to calculate the cannon muzzle velocity, pressure, and acceleration – essential factors for accurate ballistics.

Section IV presents the results derived from the EBM calculations, further visualized in Section V through graphical representations of the calculated values. A critical evaluation of various models for calculating muzzle velocity is conducted in Section VI, ensuring a robust approach to our methodology.

The work of Mungan in Section VII provides insight into dimensionless velocity and acceleration curves, an indispensable tool for theoretical understanding. Section VIII bridges theory and practice by comparing our model results with empirical data from the Pollman Video Dataset 1.

We extend our analysis in Section IX with EBM calculations tailored to velocity and acceleration, followed by the development of an analytical expression in Section X for the spud (projectile) trajectory accounting for air resistance – a factor often omitted in simplified models.

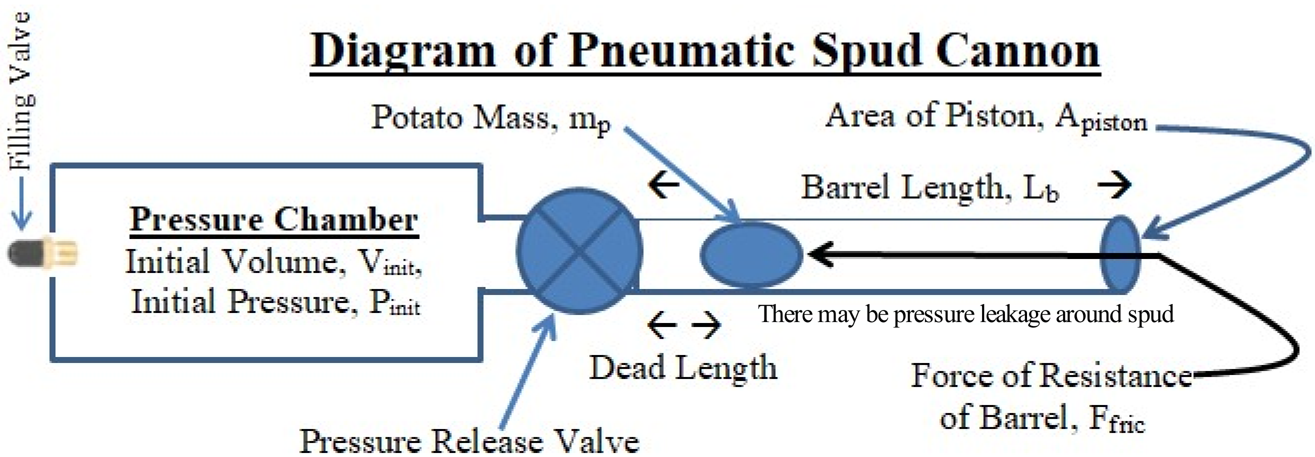
Section XI details a specially designed program that calculates trajectories at varying launch angles, allowing for a nuanced prediction of projectile paths. Finally, Section XII captures the culmination of our study with plots that trace the trajectory positions, velocities, and accelerations of projectiles, providing a visual summation of our findings.

This paper provides an interesting review of the literature concerning internal ballistics models and computational tools but also verifies these approaches against experimental data, thus offering a holistic view of internal ballistics and projectile motion.

Potato Cannon

A potato cannon is a pipe-based cannon that uses air pressure (pneumatic) to launch projectiles at high speeds. They are built to fire chunks of potato, as a hobby, or to fire other sorts of projectiles, for practical use.

These cannons have four basic components:
A filling valve, an air chamber, a pressure release valve, and a barrel.
In a pneumatic spud cannon, air is pumped into the pressure chamber. After the desired chamber pressure is reached, the pressure release valve is opened, allowing the gas to expand down the barrel, propelling the projectile forwards. The filling valve is used to pressurize the cannon and is usually a tire valve. The pressure is released with a quick release connections with ball or check valves have been used.
The pressure release valve is often one of a variety of commercially available types such as a plumbing ball valve, an irrigation sprinkler valve or a quick exhaust valve. The range of pneumatic cannons is more variable than the range of combustion spud cannons due to the increased variation possible in the components. Typical ranges are slightly higher because of the greater power, but the maximum range of some high power pneumatic cannons has been said to be over 1,000 meters (1,100 yd). Pneumatic spud cannons are generally more powerful than combustion spud cannons.



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I. Definition of Terms & Values of Muzzle & Ballistic Parameters

Given a Spud Cannon with Initial Volume, V_{init} and Pressure, P_{init}. It pushes against a piston (potato) of Diameter, D_{piston}, potato mass, m_p, with initial velocity, v_{Init}, and a Force of Friction, F_{fric}, that is constant.

Find the Final Pressure, Final Velocity, and acceleration of the piston, Δx. Assume that the air expansion is isothermal, no air leakage in the barrel, **a perfect valve that does not restrict air flow and opens instantaneously**, air pressure in the reservoir is the same as in the barrel, and the effect of atmos pressure in the barrel is negligible compared to P_{init}. We assume that the Drag Coefficient of the projectile is Cd = 1.5.

No Model for Gas Flow During the Opening of Air Valve

Compensate for **Restricted Air Flow** with a **Pressure Saturation Parameter, ξ:** $\xi := 1$

Initial Pressure $P_{initx} := 80psi\cdot\xi$ $P_{initx} = 80\cdot psi$

Chamber Initial Volume $V_{initx} := 200in^3$

Length of Barrel, Dead L_b $L_b := 40in$ $L_{dead} := 1in$

Diameter & Area of Chamber $D_{ch} := 5in$

Mass of the projectile $m_p := 0.15lb$

Gas Constants

Adiabatic Expansion: $\gamma := 1.4$

$R := 8.314\frac{J}{mol\cdot K}$

$T_{ambient} := 293K$

Size B potatoes (medium): Diameter: typically 1.5 -2.25 inches. Weight: between 5-10 ounces.

Length of Chamber $len_{ch} := V_{initx}\cdot\left[\pi\cdot(0.5\cdot D_{ch})^2\right]^{-1}$ $len_{ch} = 10.19\cdot in$

Diameter of Piston $D_{piston} := 2.016in = 0.05\,m$

Area of Piston $A_{piston} := \pi\cdot(0.5\cdot D_{piston})^2 = 2.06\times10^{-3}\,m^2$ $A_{piston} = 0.02\cdot ft^2$

Initial Velocity $v_{init} := 0.1\cdot\frac{m}{s}$

Barrel Friction $F_{fric} := 20lbf = 88.96\,N$ $P_{atm} := 14.7psi$

Piston Dead Space $P_{init} := P_{initx}\cdot 1$

Dead Volume $V_{dead} := 0in^3$ $V_{init} := V_{initx} - V_{dead}$

Muzzle Energy of 9 mm Pistol

$1150\cdot ft\cdot lbf$

Initial Energy $E_{init} := P_{init}\cdot V_{init}$ $E_{init} = 1333.3\cdot ft\cdot lbf$

Force on Piston $F_p := P_{init}\cdot A_{piston}$ $F_p = 255.36\cdot lbf$

Vol of Adiabatic Expansion, Δx $V_{final}(V_i,\Delta x) := V_i + A_{piston}\cdot\Delta x$ $V_{final}(V_{init},0.1in) = 200.32\cdot in^3$

Change in Volume $\Delta V(\Delta x) := A_{piston}\cdot\Delta x$ $\Delta V(0.1in) = 0.32\cdot in^3$

For Adiabatic Expansion $P_{final}(V_i,\Delta x) := P_{init}\cdot\left(\frac{V_i}{V_{final}(V_i,\Delta x)}\right)^\gamma$ $P_{final}(V_{init},0.1in) = 79.82\cdot psi$

Work done by gas $W_{gas}(V_i,\Delta x) := 0.5\cdot(P_{init} + P_{final}(V_i,\Delta x))\cdot\Delta V(\Delta x)$

Number of Moles $n_{moles} := \frac{P_{init}\cdot V_{init}}{R\cdot T_{ambient}}$ $n_{moles} = 0.74\,mol$ $W_{gas}(V_{init},0.1in) = 2.88\,J$

$$P_{init}\cdot\left(\frac{V_{init}}{in^3}\right)^{1.4}\cdot in^3 = 1.51\times10^4\,J$$

II. Energy Balance Model (EBM) of Incremental Distance, Δx, of Muzzle Velocity

$A_p := A_{piston}$

$$\frac{1}{2}m \cdot v_{final}^2 = \frac{1}{2}m_p \cdot v_{init}^2 + \frac{P_{init} + P_{final}(\Delta x)}{2} \cdot \Delta V(\Delta x) - F_{fric} \cdot \Delta x$$

Peak Acceleration:

$$A_{peak} := \frac{P_{init} \cdot A_p}{m_p}$$

$$A_{peak} = 1.7 \times 10^3 \cdot g$$

Find Final Velocity

$$vel_{final}(m_p, V_i, \Delta x) := \sqrt{v_{init}^2 + \frac{2}{m_p} \cdot \left(\frac{P_{init} + P_{final}(V_i, \Delta x)}{2} \cdot \Delta V(\Delta x) - F_{fric} \cdot \Delta x \right)}$$

Crude Approximation that is Not Incremental: $vel_{final}(m_p, V_{init}, 60in) = 581.3 \cdot \frac{ft}{s}$

Net Force and Acceleration

$$F_{net}(V_i, \Delta x) := \frac{P_{init} + P_{final}(V_i, \Delta x)}{2} \cdot A_{piston} - F_{fric} \qquad F_{net}(V_{init}, 0.1in) = 235.08 \cdot lbf$$

$$a(V_i, \Delta x) := \frac{F_{net}(V_i, \Delta x)}{m_p} \qquad a(V_{init}, 0.1in) = 1.57 \times 10^3 \cdot g \quad Step := \frac{L_b}{0.1in} = 400$$

$$time = \frac{a}{v} \qquad Time(V_i, \Delta x) := \frac{vel_{final}(m_p, V_i, \Delta x)}{a(V_i, \Delta x)} \qquad Time(V_{init}, 0.1in) = 5.75 \times 10^{-4} s$$

Iterative Solution

$$A_p = 2.06 \times 10^{-3} m^2$$

$$P_f(P_i, V_i, \Delta x) := P_i \cdot \left(\frac{V_i}{V_i + A_p \cdot \Delta x} \right)^{\gamma} \qquad \frac{100psi \cdot A_{piston}}{m_p} = 2.13 \times 10^3 \cdot g$$

$$P_f(P_{init}, V_{init}, 0.1in) = 79.82 \cdot psi$$

$$ai(P_i, V_i, m_p, F_{fr}, \Delta x) := \frac{1}{m_p} \left(\frac{P_i + P_f(P_i, V_i, \Delta x)}{2} \cdot A_p - F_{fr} - A_p \cdot P_{atm} \right)$$

$$ai(P_{init}, V_{init}, m_p, F_{fric}, 0.1in) = 1.25 \times 10^3 \cdot g$$

$$v_{fm}(v_i, P_i, V_i, m_p, F_{fr}, \Delta x) := \sqrt{v_i^2 + \frac{2}{m_p} \cdot \left[\frac{P_i + P_f(P_i, V_i, \Delta x)}{2} \cdot (A_p \cdot \Delta x) - F_{fr} \cdot \Delta x - A_p \cdot P_{atm} \cdot \Delta x \right]}$$

$$v_{fm}(v_{init}, P_{init}, V_{init}, m_p, F_{fric}, 0.1in) = 7.91 \frac{m}{s}$$

III. Program to Calculate Cannon Muzzle Velocity, Pressure, Acceleration

EBM: Internal Ballistic Parameters of Spud Cannon vs. Position

Calculate How the Velocity, Pressure, and Acceleration Vary Along Length of the Cannon

$$\Delta x := 0.1in \qquad v_{init} = 0.1\frac{m}{s}$$

Note: n Determines the Barrel Length -->

$$Traj(m, P_0, V_0, F_f) := \left| \begin{array}{l} P_i \leftarrow P_0 \\ V_i \leftarrow V_0 \\ v_i \leftarrow v_{init} \\ t_0 \leftarrow 0s \\ \text{for } n \in 1..710 \\ \quad \left| \begin{array}{l} v_n \leftarrow v_i \leftarrow v_{fm}(v_i, P_i, V_i, m, F_f, \Delta x) \\ P_n \leftarrow P_i \leftarrow P_f(P_i, V_i, \Delta x) \\ a_n \leftarrow ai(P_i, V_i, m, F_f, \Delta x) \\ V_i \leftarrow V_{final}(V_i, \Delta x) \\ t_n \leftarrow (v_n - v_{n-1})\cdot\dfrac{1}{a_n} + t_{n-1} \\ M_{n,0} \leftarrow v_i\cdot\dfrac{s}{ft} \\ M_{n,1} \leftarrow P_n\cdot psi^{-1} \\ M_{n,2} \leftarrow a_n\cdot g^{-1} \\ M_{n,3} \leftarrow t_n\cdot s^{-1}\cdot 10^3 \end{array} \right. \\ M \end{array} \right.$$

$$ai_m(P_i, V_i, m_p, \Delta x) := \frac{1}{m_p}\left(\frac{P_i + P_f(P_i, V_i, \Delta x)}{2}\cdot A_p - F_{fric} - A_p\cdot P_{atm}\right)$$

$$ai_m(P_{init}, V_{init}, 10oz, 0.1in) = 301.05\cdot g$$

$$v_{fm}(v_{init}, P_{init}, V_{init}, 10oz, F_{fric}, 0.1in) = 3.87\frac{m}{s}$$

IV. Results of Energy Balance Model Calculations

$V_{init} = 200 \cdot in^3$

$Vel(m,P,V,F_f) := Traj(m, P \cdot psi, V \cdot in^3, F_f \cdot lbf)^{\langle 0 \rangle} \cdot \frac{ft}{s}$

$Pres(m,P,V,F_f) := Traj(m, P \cdot psi, V \cdot in^3, F_f \cdot lbf)^{\langle 1 \rangle} \cdot psi$

$Accl(m,P,V,F_f) := Traj(m, P \cdot psi, V \cdot in^3, F_f \cdot lbf)^{\langle 2 \rangle} \cdot g$

$Velocity := Traj(m_p, P_{init}, V_{init}, F_{fric})^{\langle 0 \rangle}$

$Accel := Traj(m_p, P_{init}, V_{init}, F_{fric})^{\langle 2 \rangle}$

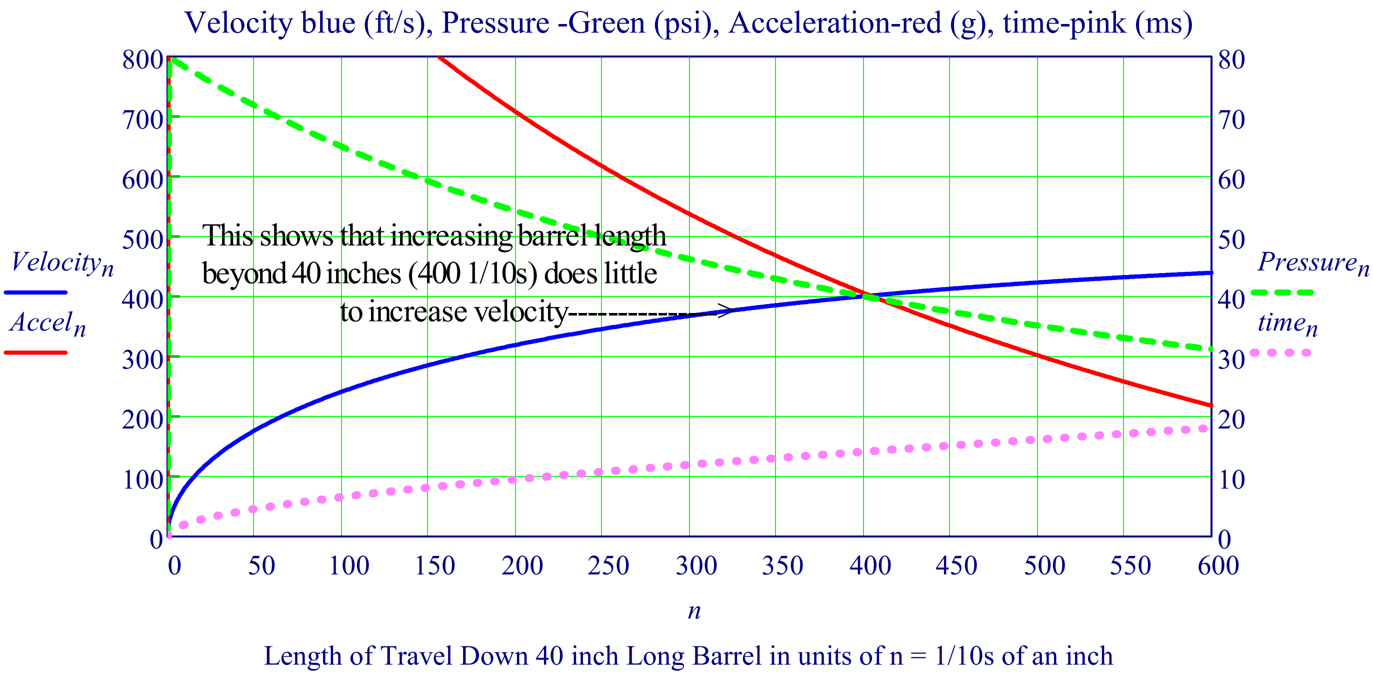
$time := Traj(m_p, P_{init}, V_{init}, F_{fric})^{\langle 3 \rangle}$

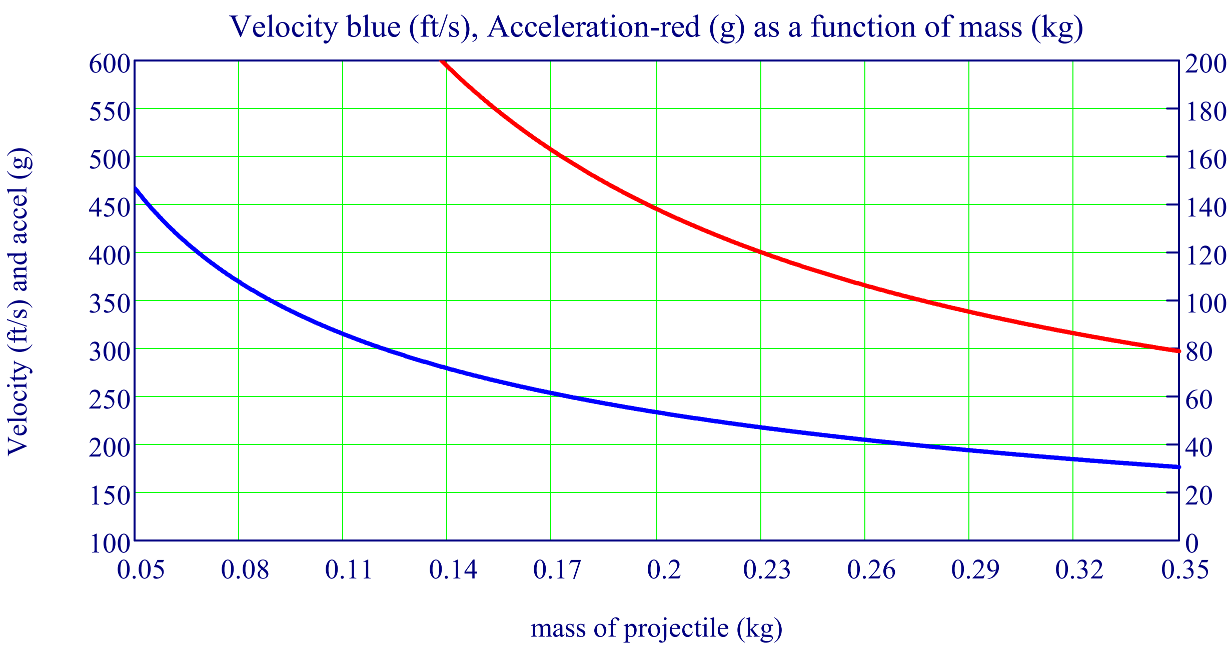
$Pressure := Traj(m_p, P_{init}, V_{init}, F_{fric})^{\langle 1 \rangle}$

$n := 0 .. 600$

IV. Plot of Energy Balance Model: Muzzle V vs. Position

48/60 inches long Barrel Driven by 80 psi Tank





VI. Comparison of Four Calculation Methodologies

Unfortunately, the cited reference reports that none of these 4 Methods give very accurate results.

Compressed-Air Spud Cannon and Performance Assessment - Crawford

m Mass of the projectile, kg $\quad m_p = 0.15 \cdot lb$

Po Initial pressure in the air reservoir $\quad P_0 := P_{init} = 80 \cdot psi$

Vo Air reservoir volume, m^3 $\quad V_0 := V_{init} = 200 \cdot in^3$

A Cross-sectional area of the barrel $\quad A := A_{piston} = 0.0021\, m^2$

Length $\quad L_b := 60in$

P_{atm} $\quad P_{atm} = 14.7 \cdot psi$

Friction $\quad f := F_{fric} = 20 \cdot lbf$

$\gamma = 1.4$

$\gamma_{iso} := 0.4$

Model 1:
Isothermal

$$V_1(L_b) := \sqrt{\frac{5P_0 \cdot V_0}{m_p} \cdot \left[1 - \left(\frac{V_0}{A \cdot L_b + V_0}\right)^{\gamma_{iso}}\right] - \frac{2A \cdot L_b \cdot P_{atm}}{m_p} - \frac{2 \cdot L_b \cdot f}{m_p}} \qquad V_1(L_b) = 439.75 \cdot \frac{ft}{s}$$

Equation $V_2(L_b)$: *Projectile Velocity of an Air Cannon*, Rohrbach Model

Model 2:
Rohrbach
Isothermal

$$V_2(L_b) := \sqrt{\frac{2}{m_p} \cdot \left(P_0 \cdot V_0 \cdot \log\left(1 + \frac{A \cdot L_b}{V_0}, e\right) - A \cdot L_b \cdot P_{atm} - L_b \cdot f\right)} \qquad V_2(L_b) = 490.58 \cdot \frac{ft}{s}$$

Model 3:

Spudgun Technology Center (STC) Excel Model:

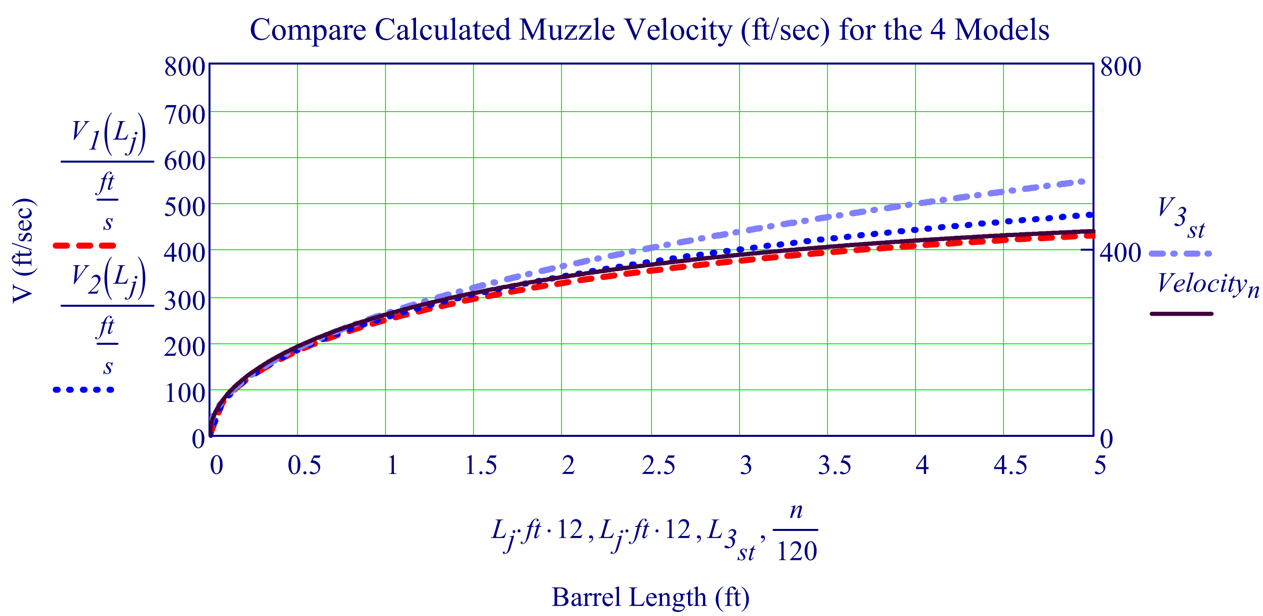
Read Data from STC Model $\quad STC := READPRN(\text{"SGTech.txt"}) \quad st := 0 .. 999$

$$V_3 := STC^{\langle 5 \rangle} \cdot \frac{m}{ft} \qquad L_{3_{st}} := \frac{L_b \cdot st}{999} \cdot \frac{m}{ft} \qquad P_3 := STC^{\langle 1 \rangle} \qquad V_{3_{998}} \cdot \frac{ft}{s} = 549.18 \cdot \frac{ft}{s}$$

Model 4:
Energy Balance Model

$Velocity_n$ is the Velocity Calculated from Conservation of Energy on Page 2

$$Length := 600 \cdot 0.1in = 5 \cdot ft \qquad Velocity_{600} \cdot \frac{ft}{s} = 439.75 \cdot \frac{ft}{s}$$

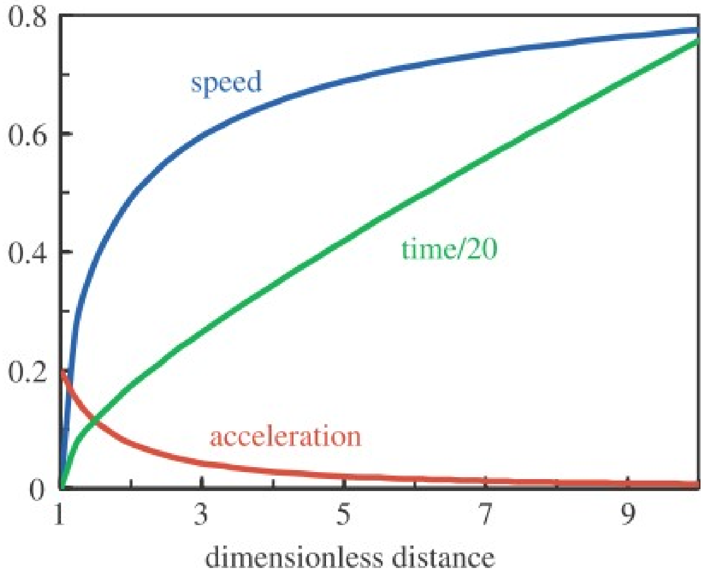


VII. Dimensionless Velocity & Acceleration Curves - Mungan Paper

Use EBM Model Match These Sim Curves

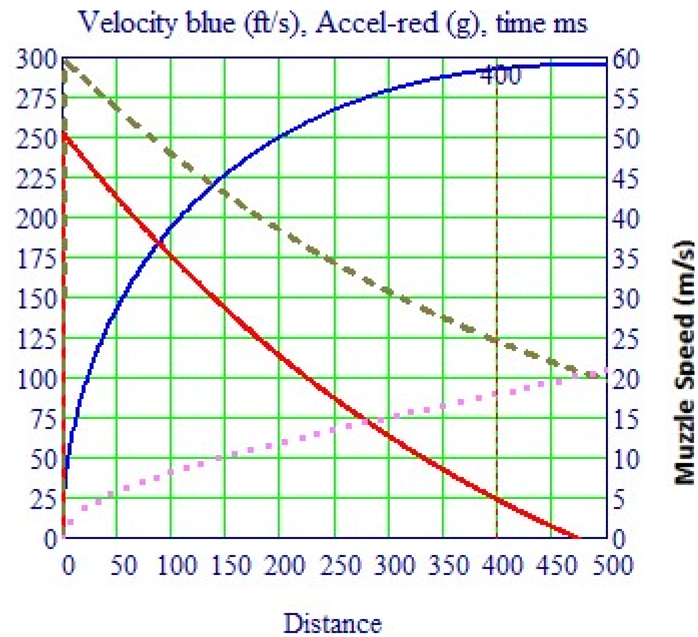
Internal ballistics of a pneumatic potato cannon,

Carl E Mungan, Eur. J. Phys. 30 (2009) 453–457



Spud Cannon From Page 3

It has the **Same Shape** as the Mungan Curves



Dimensionless speed, acceleration and time as a function of distance for a projectile propelled along a cylinder by a diatomic gas. The time has been divided by 20 to keep it on scale.

VIII. Compare Velocity from this Sim to Data from Pollman Video Measurements

Source of Experimental Data: ***Video measurement of the muzzle velocity of a potato gun,*** **Pollman**

Test Conditions for Pollman Pneumatic Cannon Video Muzzle Velocity Paper

Gauge Pressure from 45 to 120 psi, mass from 0.05 to 0.35 kg, $\mathbf{Vol_{tank}}$ **= 9040 mL,**

Barrel 5.08 cm (2 inches) x 1.8 m (71in) long, Volume = 3640 ml (5.08 cm x 1.8m long). Mass from 0.05 to 0.35 kg, Kinetic Friction Force Estimate: F_{fric} 1 to 2N n varies from 1.5 to 3.38 mol

Pollman Tank Volume: $Vol_{tank} := 9040mL = 551.65 \cdot in^3$ $V_{ini} := Vol_{tank}$ $2N = 0.45 \cdot lbf$ $0.1kg = 0.22 \cdot lb$

Digitize (https://automeris.io/WebPlotDigitizer/) Data Points from Pollman's Data Points

$V_{120} := READPRN(\text{"Video Pollman red.csv"})$

$V_{110} := READPRN(\text{"Video Pollman black.csv"})$

$V_{100} := READPRN(\text{"Video Pollman green.csv"})$

$V_{80} := READPRN(\text{"Video Pollman BlueTT.csv"})$

$V_{45} := READPRN(\text{"Video Pollman Violet2.csv"})$

71 inch long barrel => 710th Row for Velocitym Function $n := 0..720$

Velocity Model for mass, Pressure, and Muzzle Resistance

$$Vel\left(m_p, \frac{P_{init}}{psi}, \frac{V_{ini}}{in^3}, \frac{F_{fric}}{lbf}\right)_{400} = 468.47 \cdot \frac{ft}{s}$$

$$Accl\left(m_p, \frac{P_{init}}{psi}, \frac{V_{ini}}{in^3}, \frac{F_{fric}}{lbf}\right)_{400} = 825.42 \cdot g$$

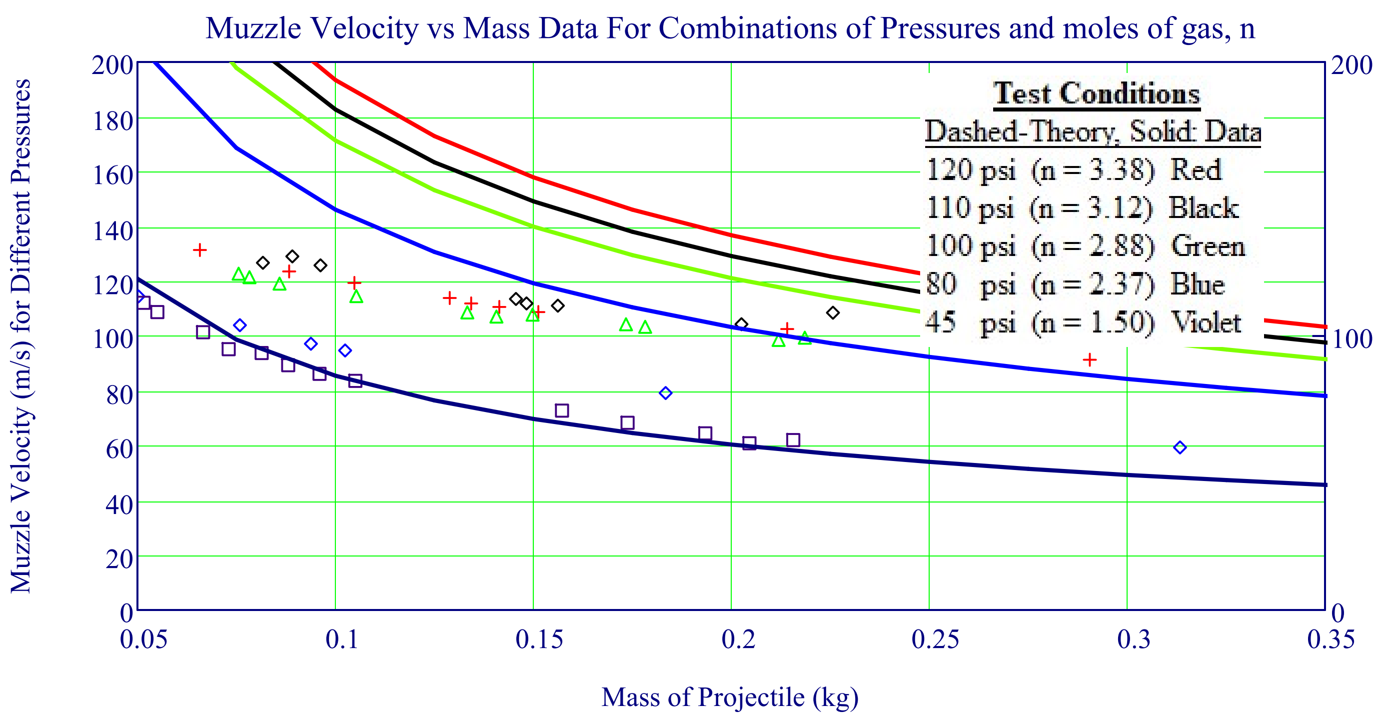
$$Pres\left(m_p, \frac{P_{init}}{psi}, \frac{V_{ini}}{in^3}, \frac{F_{fric}}{lbf}\right)_{400} = 59.77 \cdot psi$$

$$Velc(mp, P) := Vel\left(mp \cdot kg, P, \frac{V_{ini}}{in^3}, \frac{F_{fric}}{lbf}\right)_{710}$$

$jj := 0, 1..15$ $mp_{jj} := 0.025 \cdot jj + 0.025$

Data: Video measurement of the muzzle velocity (m/s) of a potato gun, Anthony Pollman

The experimental data has a large amount of scatter. This plot is a crude approximation at best.



Mark D's Simulation Parameters

$P_{init} = 80 \cdot psi$

$m_p = 0.07\,kg$

$n_{old} := \frac{P_{init} \cdot V_{init}}{R \cdot T} = 217.44 \frac{s^2 \cdot K \cdot A \cdot mol}{kg}$

$Velocity_{346} \cdot \frac{ft}{s} = 117.2 \frac{m}{s}$

Velocity, Pressure, Mass for a 40 inch Barrel for M. Decker's Cannon:

$Velocity_{400} \cdot \frac{ft}{s} = 400.71 \cdot \frac{ft}{s}$ $P_{init} = 551.58 \cdot kPa$ $P_{init} = 80 \cdot psi$ $m_p = 0.07\,kg$ $m_p = 0.15 \cdot lb$

$V_{initx} := 2.95L$

Length of Barrel for Video Test Data: $Len_{vid} := 88cm = 34.65 \cdot in$

Find Volume to Match Pollman Video Paper

From Chart Below: $n_{new} := 2.37$ $V_{n288} := \frac{n_{new} \cdot R \cdot T}{P_{init}}$ $V_{n288} = 2.18 \frac{kg}{s^2 \cdot K \cdot A} \cdot \frac{in^3}{mol}$

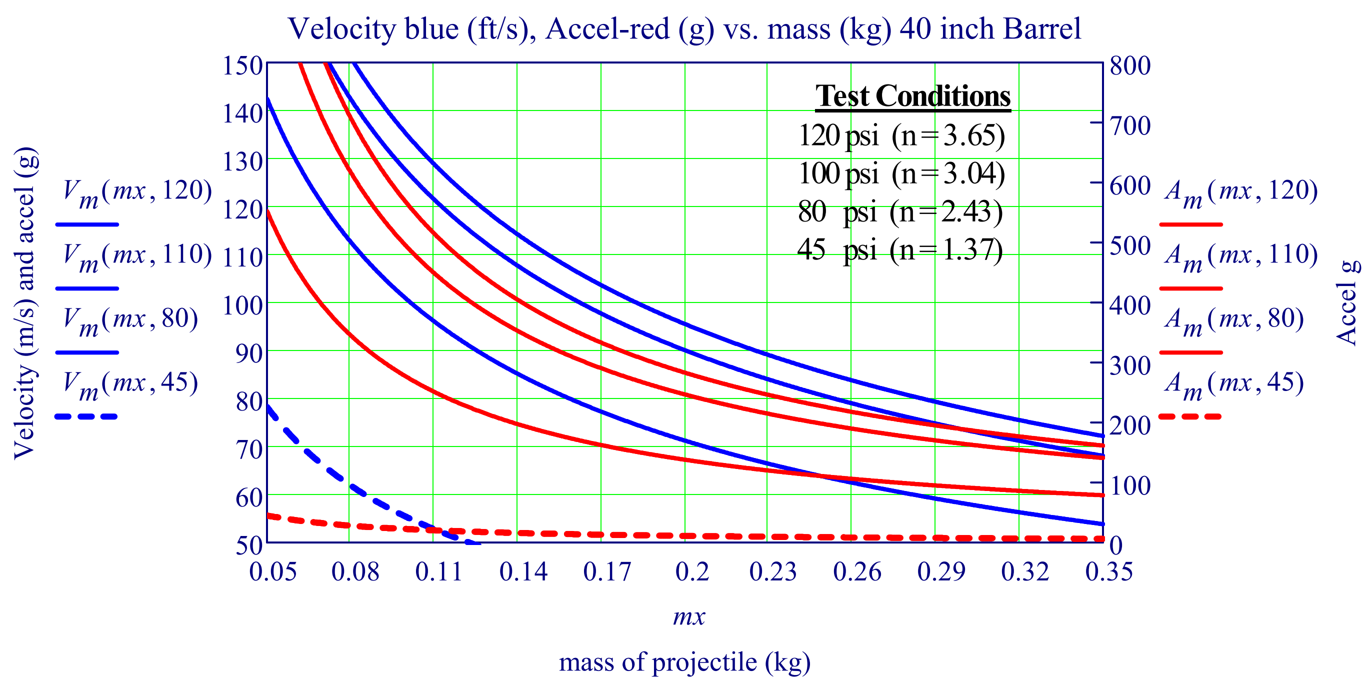
IX. Model Calculations for Velocity and Acceleration from Page 2 and 3
Velocity and Acceleration vs. Mass for Various Chamber Pressures

Definitions of Vm and Am are not shown. They are just the previous v and P Equations with m and P as variables.

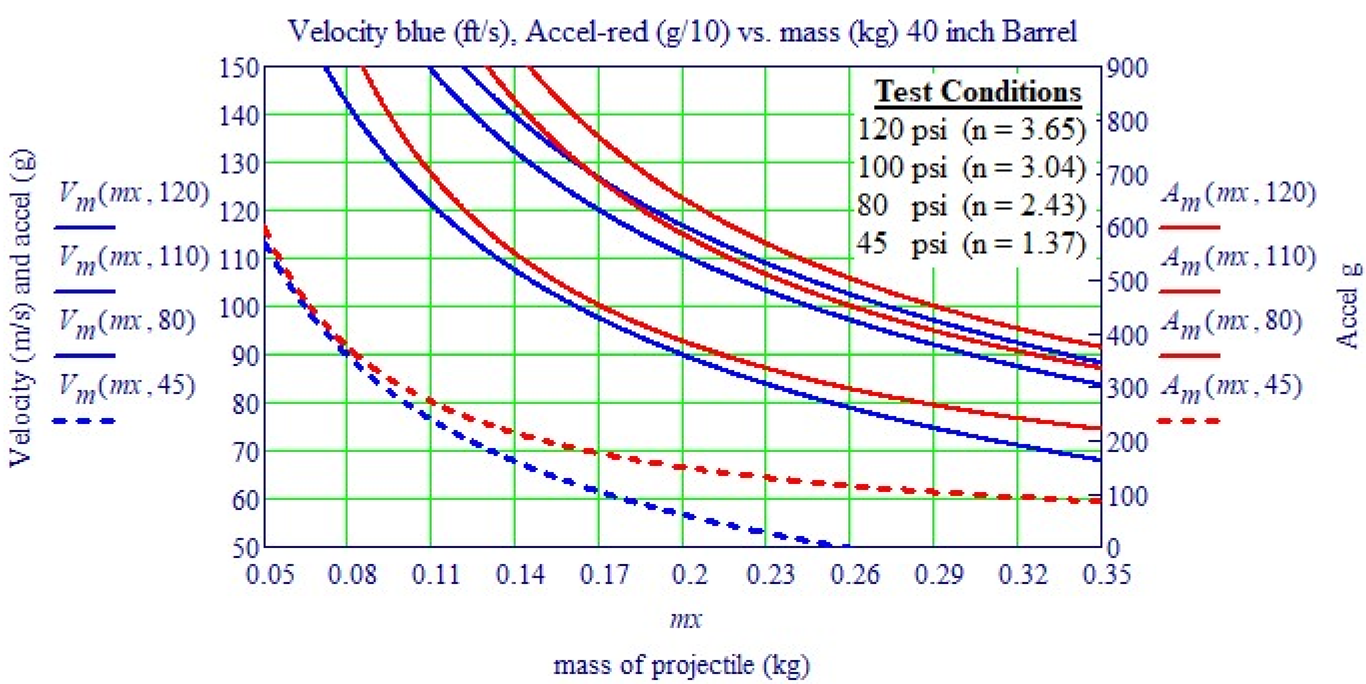
$$V_m(mx, P) := Vel\left(mx \cdot kg, P, \frac{V_{init}}{in^3}, \frac{F_{fric}}{lbf}\right)400 \qquad A_m(mx, P) := Accl\left(mx \cdot kg, P, \frac{V_{init}}{in^3}, \frac{F_{fric}}{lbf}\right)400\frac{1}{g}$$

NOTE: The number of moles for this Model was adjusted to approximately match the number moles in test data

Adiabatic Plot: γ = 1.4



Isothermal Plot: γ = 0.4

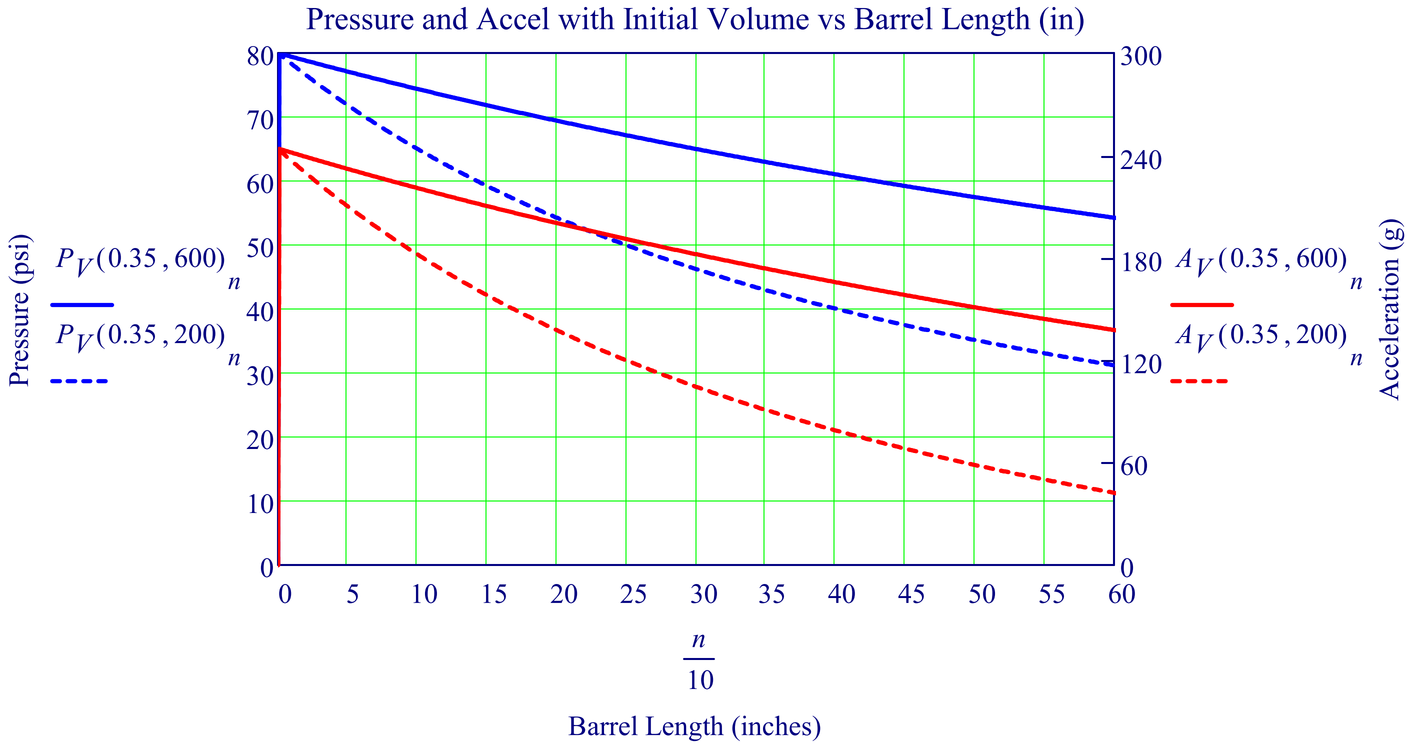


X. Variation of Pressure and Acceleration vs. Initial Chamber Volume

$$P_V(mx, V) := Pres\left(mx \cdot kg, 80, V, \frac{F_{fric}}{lbf}\right)\frac{1}{psi} \qquad A_V(mx, V) := Accl\left(mx \cdot kg, 80, V, \frac{F_{fric}}{lbf}\right)\frac{1}{g}$$

$$P_V(0.35, 200)_3 = 79.47 \qquad A_V(0.35, 200)_1 = 243.11$$

Initial Pressure = 80 psi, Mass of Spud = 0.35kg (0.77 lbs)



X. Analytic Expression for the Spud Trajectory for Initial Velocity

The trajectory equation for a projectile fired at an angle θ with initial velocity, vinit, can be obtained from the parametric equations of motion under constant acceleration due to gravity. For an angle of 45 degrees, these equations become:

Note: The Final Muzzle Velocity is the Initial Velocity for the Beginning of the Trajectory

Final Muzzle Velocity $v_{init} := Velocity_{400} \cdot \frac{ft}{s}$ $v_{init} = 400.71 \cdot \frac{ft}{s}$ $v_{init} = 122.14 \frac{m}{s}$

x direction is distance $x(t) := v_{init} \cdot t \cdot cos(45deg)$

y direction is up and down $y(t) := v_{init} \cdot t \cdot sin(45deg) - \frac{1}{2} g \cdot t^2$

$cos(45deg) = 0.71$ $sin(45deg) = 0.71$

$x(1sec) = 283.35 \cdot ft$ $y(1sec) = 81.46 m$

To eliminate time t and get y as a function of x, we solve for t from the x(t) equation and substitute into the y(t) equation

$$time(x, \theta) = \frac{x(t)}{v_{init} \cdot cos(\theta deg)}$$

$$y(x, \theta) := v_{init} \cdot sin(\theta \cdot deg) \cdot \left(\frac{x}{v_{init} \cdot cos(\theta \cdot deg)} \right) - \frac{1}{2} g \cdot \left(\frac{x}{v_{init} \cdot cos(\theta \cdot deg)} \right)^2$$

Calculate the Cannon Trajectory for a Spud with Air Drag

Note: The Final Muzzle Velocity is the Initial Velocity for the Beginning of the Trajectory

$\Delta t := 0.01 \cdot s$

Drag Coefficient $Cd := 1.5$

Density of Air $\rho_{air} := 1.293 \cdot \frac{gm}{L}$

Drag Force for Frontal Area of Spud $D_{drag} := 0.5 \cdot \rho_{air} \cdot A_{piston} \cdot Cd$

Final Muzzle Velocity: $vmuz_{final} := Velocity_{600} \cdot \frac{ft}{s}$ $vmuz_{final} = 134.04 \frac{m}{s}$

Initial Trajectory Velocities: $v_{0x}(v_{muz}, \theta) := v_{muz} \cdot cos(\theta \cdot deg)$ $v_{0y}(v_{muz}, \theta) := v_{muz} \cdot sin(\theta \cdot deg)$

XI. Program to Calculate Trajectories for Different Launch Angles

Given the Final Muzzle Velocity, v_{muz}

$$nn := 600 \qquad i := 1..nn \qquad x_0 := 0\frac{ft}{s} \qquad y_0 := 0\frac{ft}{s}$$

Note: No barrel friction after launch

$$
\begin{aligned}
Tr(v_{muz},\theta) := \; & y_0 \leftarrow x_0 \leftarrow x_0 \\
& v_x \leftarrow v_{0x}(v_{muz},\theta) \\
& v_y \leftarrow v_{0y}(v_{muz},\theta) \\
& \text{for } i \in 1..nn \\
& \quad v \leftarrow \sqrt{v_x^2 + v_y^2} \\
& \quad a_x \leftarrow \frac{-D_{drag}\cdot v\cdot v_x}{m_p} \\
& \quad a_y \leftarrow -g - \frac{D_{drag}\cdot v\cdot v_y}{m_p} \\
& \quad x_i \leftarrow x_{i-1} + v_x\cdot \Delta t + 0.5\cdot a_x\cdot \Delta t^2 \\
& \quad y_i \leftarrow y_{i-1} + v_y\cdot \Delta t + 0.5\cdot a_y\cdot \Delta t^2 \\
& \quad v_x \leftarrow v_x + a_x\cdot \Delta t \\
& \quad v_y \leftarrow v_y + a_y\cdot \Delta t \\
& \quad M_{i,0} \leftarrow \frac{x_i}{ft} \\
& \quad M_{i,1} \leftarrow \frac{y_i}{ft} \\
& \quad M_{i,2} \leftarrow v_x\cdot\frac{s}{ft} \\
& \quad M_{i,3} \leftarrow v_y\cdot\frac{s}{ft} \\
& \quad M_{i,4} \leftarrow a_x\cdot g^{-1} \\
& \quad M_{i,5} \leftarrow a_x\cdot g^{-1} \\
& M
\end{aligned}
$$

$$\frac{-D_{drag}\cdot vmuz_{final}^2}{m_p} = -527.34\frac{m}{s^2}$$

$$-g - \frac{D_{drag}\cdot vmuz_{final}^2}{m_p} = -537.14\frac{m}{s^2}$$

XIII. Plots of Trajectory Positions, Velocities, and Accelerations

NOTE: Air Drag reduces Spud Range and Final Velocity by 56%

Vertical Axis is Height in Feet. Horizontal Axis is Distance in Feet

Given Initial Pressure of 80 psi, Volume of 200 in^3, and Barrel Length 40 inches, the final muzzle velocity is:

Final Muzzle Velocity: $v_{muz} := 400 \cdot \frac{ft}{s}$

Velocity in X Direction

$X(\theta) := Tr(v_{muz}, \theta)^{\langle 0 \rangle}$

$Y(\theta) := Tr(v_{muz}, \theta)^{\langle 1 \rangle}$

$VX(\theta) := Tr(v_{muz}, \theta)^{\langle 2 \rangle}$

$VY(\theta) := Tr(v_{muz}, \theta)^{\langle 3 \rangle}$

$v_{total}(\theta) := \sqrt{VX(\theta)^2 + VY(\theta)^2}$

Acceleration in X Direction

$AX(\theta) := Tr(v_{muz}, \theta)^{\langle 4 \rangle}$

$Time_i := i \cdot \Delta t \cdot 10$

$VX(30)_{400} \cdot \frac{ft}{s} = 13.35 \cdot mph$

$v_{total}(30)_1 \cdot \frac{ft}{s} = 262.86 \cdot mph$

$v_{total}(30)_{400} \cdot \frac{ft}{s} = 35.37 \cdot mph$